

# **Systèmes dynamiques**

**Actionneurs et systèmes électromagnétiques**

# Équations des systèmes électromécaniques

EPFL

$$u_j = R_j i_j + \frac{d\Psi_j}{dt} \quad (k \text{ circuits})$$

$$\Psi_j = \sum_{p=1}^k L_{jp} i_p$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2}$$

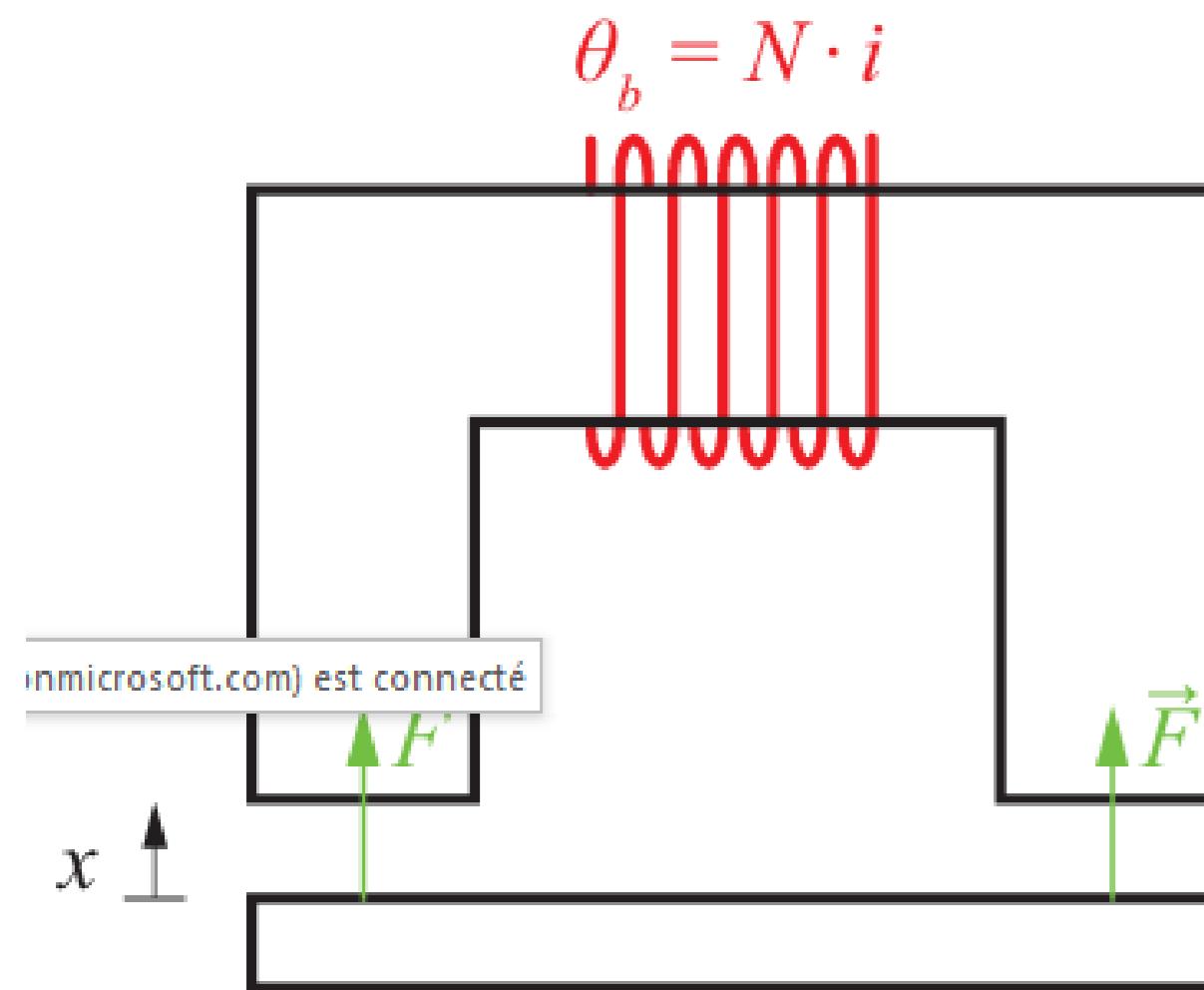
$$\sum M_m = J_m \frac{d^2 \alpha_m}{dt^2}$$

(n degrés de liberté)

$$F_m = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{d\Lambda_{jp}}{dx_m} \Theta_j \Theta_p$$

$$M_m = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{d\Lambda_{jp}}{d\alpha_m} \Theta_j \Theta_p$$

# Exemple: système réluctant



Cas linéaire:

$$u = Ri + L \frac{di}{dt} + i \frac{dL}{dx} \frac{dx}{dt}$$

$$u = Ri + L \frac{di}{dt} + \frac{2Fv}{i}$$

$$\varphi = L i$$

$$u_j = R_j i_j + \frac{d\Psi_j}{dt} \text{ avec } \Psi_j = \sum_{p=1}^k L_{jp} i_p$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2}$$

$$F_x = \frac{1}{2} \frac{d\Lambda_a}{dx} \Theta_a^2 + \frac{d\Lambda_{ab}}{dx} \Theta_a \Theta_b + \frac{1}{2} \frac{d\Lambda_b}{dx} \Theta_b^2$$

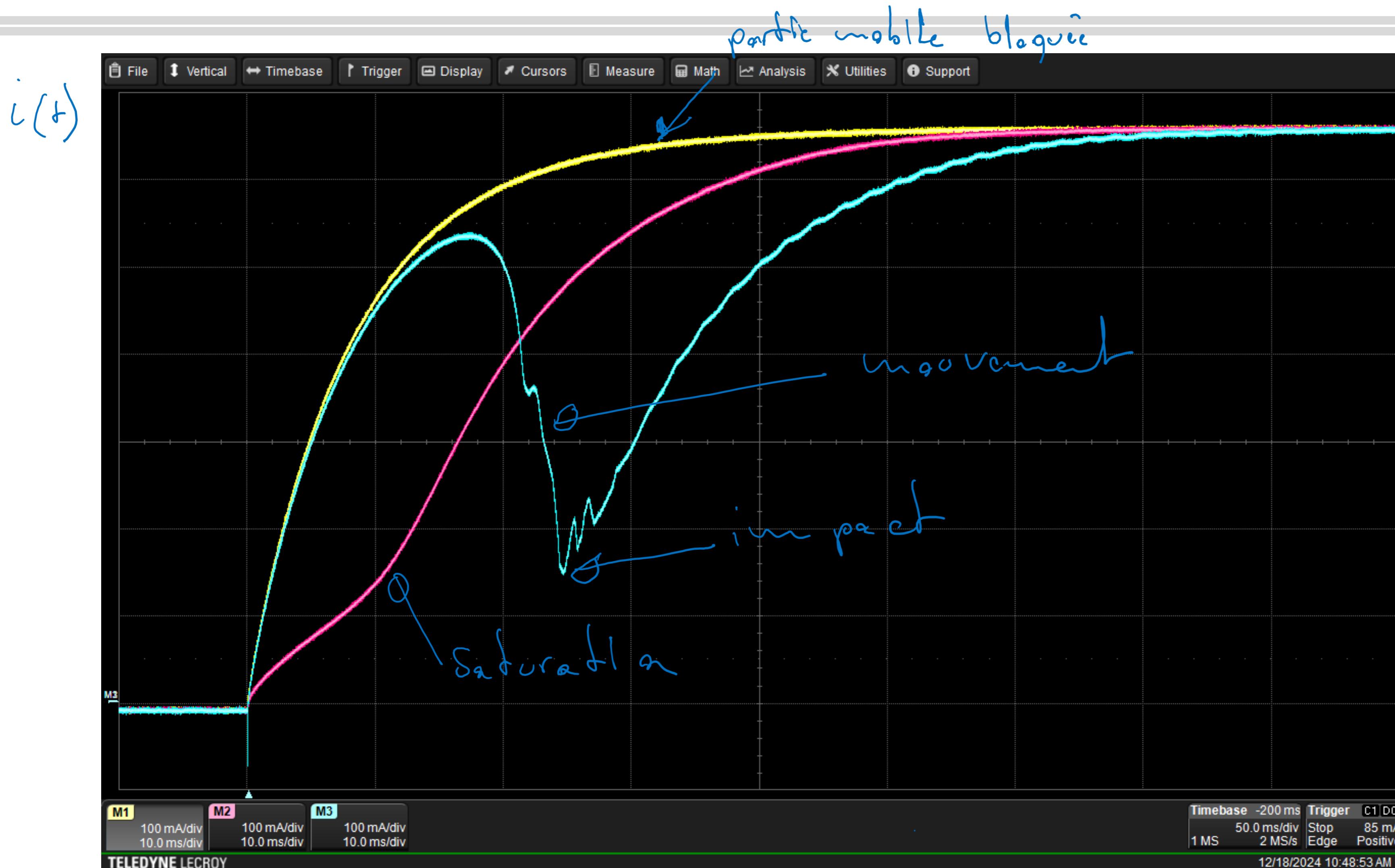
$$F_x = \frac{1}{2} \frac{d\Lambda_b}{dx} \Theta_b^2 = \frac{1}{2} \frac{dL}{dx} i^2 = m \frac{d^2 x}{dt^2}$$

$$u = Ri + L \frac{di}{dt} + i \frac{dL}{dt} = Ri + L \frac{di}{dt} + i \underbrace{\frac{\partial L}{\partial x} \frac{dx}{dt}}_{\text{transformation}} + i \underbrace{\frac{\partial L}{\partial i} \frac{di}{dt}}_{\text{saturatie}}$$

$$L = \frac{N^2 \mu S}{l}$$

tension induite

# Exemple système réluctant (mesures)



# Exemple: système polarisé

$$u_j = R_j i_j + \frac{d\Psi_j}{dt}$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2}$$

$$F_x = \frac{1}{2} \frac{d\Lambda_a}{dx} \Theta_a^2 + \frac{d\Lambda_{ab}}{dx} \Theta_a \Theta_b + \frac{1}{2} \frac{d\Lambda_b}{dx} \Theta_b^2$$

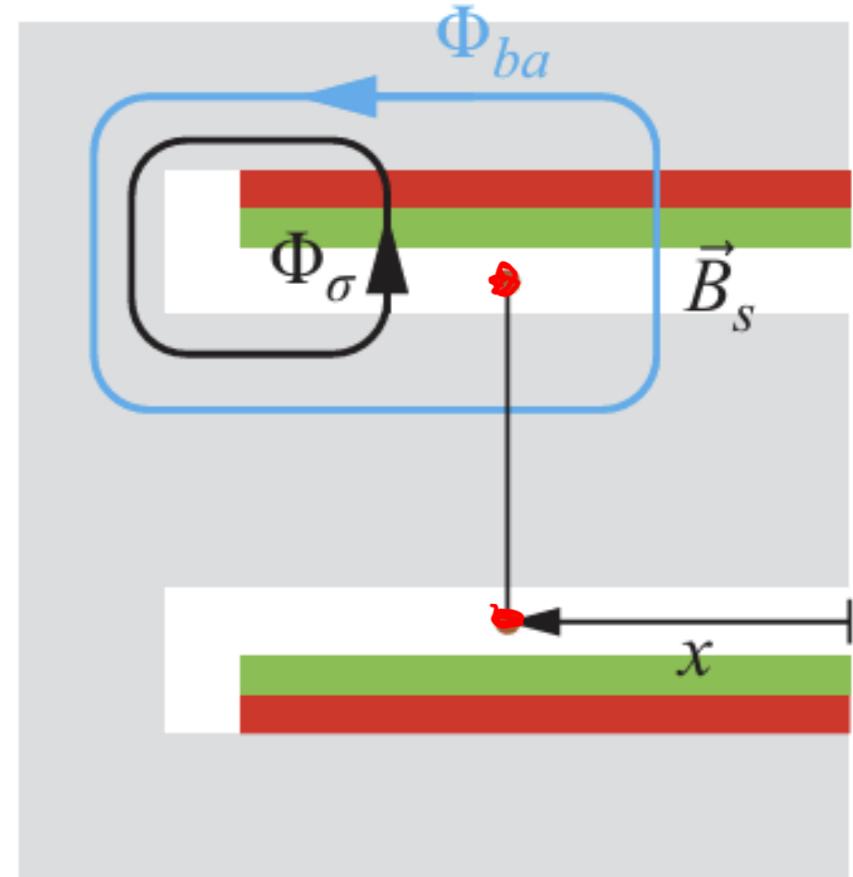
$$F_x = \frac{d\Lambda_{ab}}{dx} \Theta_a \Theta_b = \frac{d\Lambda_{ab}}{dx} \Theta_a N_i = K_i$$

$$u = Ri + \frac{d\Psi}{dt}$$

avec:

$$\Psi = Li + \Psi_{ba} = Li + N\Lambda_{ba}\Theta_a$$

$$u = Ri + L \frac{di}{dt} + \frac{d\Psi_{ba}}{dt} = Ri + L \frac{di}{dt} + N\Theta_a \frac{d\Lambda_{ba}}{dx} \frac{dx}{dt} = R_i + L \frac{\dot{i}}{dt} + K_i$$



# Résolution

$$u = Ri + L \frac{di}{dt} + k\nu$$

$$F = m \frac{d^2x}{dt^2} = m \frac{dv}{dt}$$

$$F = ki$$

$$i = \frac{m}{k} \frac{dv}{dt}$$

$$u = R \frac{m}{k} \frac{dv}{dt} + L \frac{m}{k} \frac{d^2v}{dt^2} + k\nu$$

ou encore:

$$\frac{d^2v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{k^2}{mL} v = u \frac{k}{mL}$$

